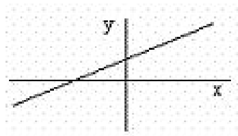
Sample Mathematics Placement Test Questions

The Mathematics Placement Test used at Columbia College consists of 32 multiple choice questions that cover basic knowledge of arithmetic, elementary algebra, trigonometry, logarithms and exponential functions. Calculators of any kind are not allowed for this test. A good reference book for these topics is the textbook currently used in Mathematics 100: Precalculus: Functions and Graphs by Swokowski and Cole.

Sample Questions.

1. Simplify $\frac{99}{117}$.
2. Subtract 6.478 from 731.35.
3. Add $\frac{3}{48}$ to $\frac{7}{60}$.
4. Divide 63.6 by 0.0012.
5. Express $\frac{241}{80}$ in decimal form.
6. Divide $\frac{7}{15}$ by $\frac{21}{120}$.
7. Find 70% of a number x given that 15% of x is 38.
8. Find the sum of 728 and 274.
9. Find the integer that is closest to $\sqrt{40018}$.
10. Solve for x: $5(3x - 2) - 7(2x + 8) = 13x + 3$.
11. Factor $6x^2 - 5x - 6$.
12. Add and simplify: $\frac{x}{x-2} - \frac{8}{x^2-4}$
13. Simplify $\frac{5 + \frac{x}{x+2}}{3 - \frac{1}{x+2}}$
14. Simplify and express without negative exponents: $\frac{(2x^{-3}y^4)^3}{4x^2y^{-3}}$

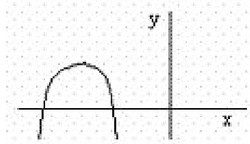
15. Which of the following equations has a graph similar to the one shown at the right?



A) $y = x+2$ B) $y = 2x+2$ C) $y = \frac{1}{2}x + 2$ D) $y = -\frac{1}{2}x - 2$

E) $y = \frac{1}{2}x - 2$

16. Which of the following equations has a graph similar to the parabola shown at the right?



A) $y = -(x+2)^2 - 1$ B) $y = -(x+2)^2 + 1$

C) $y = (x-2)^2 + 1$ D) $y = (x+2)^2 + 1$ E) $y = (x+2)^2 - 1$

17. Express the complex number $(3+2i)(4-i) + \frac{5+5i}{2+i}$ in standard form.

18. Solve the system of equations:

$$2x + 3y = 47$$
$$5x - 4y = -9$$

19. Let $f(x) = 13x+7$ and let $f^{-1}(x)$ be the inverse of f(x). Find $f^{-1}(98)$.

20. If $f(x) = x^2 + 5x - 8$ and $g(x) = x^2 + 3x + 1$, express the composite function f(g(x)) is standard form.

21. Calculate $\log_2\left(\frac{8}{\sqrt[3]{2}}\right)$

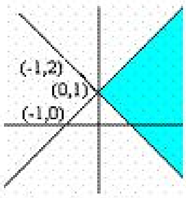
22. Express $\log\left(\frac{x^3\sqrt{y}}{z^2}\right)$ in terms of logx, logy and logz.

23. Given $y = \frac{2x+1}{3x-1}$, express x in terms of y.

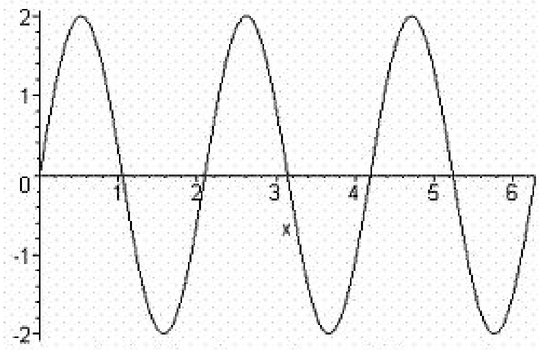
24. Solve $2(x + 5) - 3(3x - 2) < 5x + 2$.

25. Approximate sin(132?) to one decimal place.

26. Given a right triangle ABC where $\angle ABC$=90?, AC = 10 and BC = 6, find the secant of $\angle BAC$.



27. Using set notation, describe the shaded region shown above.



28. The graph of $y = a\sin(bx)$ is shown above. What are the values of a and b?

29. If $\sin\theta = \dfrac{2}{3}$ and $\dfrac{\pi}{2} < \theta < \pi$, find $\tan\theta$.

30. Which of the following equations have straight lines as their graphs?
I) $x = 3$ II) $x^2 - y^2 = 1$ III) $3x + 2y = 8$ IV) $x^2 + y^2 = 1$ V) $y = 0$

31. If $3^{x+2} = 18$, express x as a logarithm.

32. Solve the inequality: $x^5 + x^4 - 2x^3 < 0$.

Answers

1. $\dfrac{99}{117} = \dfrac{9\times 11}{9\times 13} = \dfrac{11}{13}$. A fraction is simplified by dividing the top and the bottom by the greatest common factor of the top and bottom - which in this case is 9.

2.

$$\begin{array}{r} 731.350 \\ \underline{6.478} \\ 724.872 \end{array}$$

When adding or subtracting numbers in decimal form you must "line up" the decimal points.

3. $\dfrac{3}{48} + \dfrac{7}{60} = \dfrac{3\times 5}{48\times 5} + \dfrac{7\times 4}{60\times 4} = \dfrac{15}{240} + \dfrac{28}{240} = \dfrac{43}{240}$. To add or subtract fractions you must convert the fractions to equivalent fractions that have a common denominator – in this case 240. The best choice of common denominator is the least common multiple of the two denominators – in this case, since $48 = 4\times 12$ and $60 = 5\times 12$, the lowest common multiple is $4\times 5\times 12 = 240$

4. $\frac{63.6}{0.0012} = \frac{636000}{12} = 53000$. When dividing numbers in decimal form, first multiply the top (dividend) and the bottom (divisor) by the smallest power of 10 – in this case 10000 – that makes the bottom an integer and then proceed with ordinary long division.

5.

```
       3.0125
80 ) 241.0000
     240
       100
        80
        200
        160
         400
         400
           0
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6. $\frac{7}{15} \div \frac{21}{120} = \frac{7}{15} \times \frac{120}{21} = \frac{7 \times 8 \times 15}{15 \times 3 \times 7} = \frac{8}{3}$. Dividing a number by a fraction $\frac{a}{b}$ is equivalent to multiplying the number by the reciprocal $\frac{b}{a}$ (first step) The numerator (top) of the product of fractions is the product of their numerators and the denominator of a product of fractions is the product of their denominators (second step). A fraction is simplified by dividing the numerator and denominator by their greatest common factor – in this case $15 \times 7 = 105$.

7. "15% of x" means $\frac{15}{100}x$. Therefore $\frac{15}{100}x = 30$, $15x = 3000$, $x = 200$.

Therefore 70% of the number $= \frac{70}{100} \times 200 = 140$

8. The sum of the digits in the units column is 8+4 =12 which means that a 1 is "carried over" to the tens column: 1+2+7 = 10 and a 1 is carried over to the hundreds column: 1+7+2 = 10

$$\begin{array}{r} 728 \\ 274 \\ \hline 1002 \end{array}$$

9. Since $\sqrt{40000} = \sqrt{4} \times \sqrt{10000} = 2 \times 100 = 200$, 200 is a good approximation of $\sqrt{40018}$ and since $201^2 = (200+1)^2 = 200^2 + 2 \times 200 + 1^2$ $= 40401$, 200 appears to be the closest integer to $\sqrt{40018}$ and in fact is the closest integer.

10. $5(3x - 2) - 7(2x + 8) = 13x + 3$

$15x - 10 - 14x - 56 = 13x + 3$ (expanded both sides)

$x - 66 = 13x + 3$ (collected like terms)

$-12x = 69$ (Subtracted 13x from both sides and added 66 to both sides)

$x = -\frac{69}{12} = -\frac{33}{4}$ (divided both sides by -12 and then simplified)

11. $6x^2 - 5x - 6 = (3x + 2)(2x - 3)$. This can be done by trial and error by inserting integers for a, b, c and d in (ax+b)(cx+d) or by using the quadratic formula to find the roots of $6x^2 - 5x - 6=0$ and then using the Factor theorem: $6x^2 - 5x - 6=0$ if $x = \frac{5 \pm \sqrt{25+144}}{12} = \frac{5 \pm 13}{12} = \frac{3}{2}$ or $-\frac{2}{3}$.

Therefore $6x^2 - 5x + 6 = 6(x + \frac{2}{3})(x - \frac{3}{2}) = (3x + 2)(2x - 3)$.

12. $\frac{x}{x-2} - \frac{8}{x^2 - 4} = \frac{x(x+2) - 8}{(x-2)(x+2)} = \frac{x^2 + 2x - 8}{(x-2)(x+2)} = \frac{(x-2)(x+4)}{(x-2)(x+2)} = \frac{x+4}{x+2}$

13. Multiply the top and bottom by x+2 to get $\frac{5x + 10 + x}{3x + 6 - 1} = \frac{6x + 10}{3x + 5} = 2$

14. $\frac{(2x^{-3}y^4)^3}{4x^2y^{-3}} = \frac{8x^{-9}y^{12}}{4x^2y^{-3}} = \frac{2y^{15}}{x^{11}}$. The following rules of exponents were used: $(xyz)^n = x^n y^n z^n$, $(x^a)^b = x^{ab}$, $x^{-n} = \frac{1}{x^n}$ and $\frac{x^a}{x^b} = x^{a-b}$.

15. The slope of the line is approximately $\frac{1}{2}$ and the y-intercept is positive. Therefore C) is the best choice.

16. Since the parabola "opens down" it is the translation to the left and up of a parabola of the form $y = ax^2$ where a is a negative number. Therefore B) is the best choice.

17. $(3+2i)(4-i) = 12 - 3i + 8i - 2i^2 = 14 + 5i$ (because $i^2 = -1$)

$\frac{5+5i}{2+i} = \frac{(5+5i)(2-i)}{(2+i)(2-i)} = \frac{10 - 5i + 10i - 5i^2}{4 - i^2} = \frac{15 + 5i}{5} = 3 + i$.

Therefore $(3+2i)(4-i)+\frac{5+5i}{2+i}=(14+5i)+(3+i)=17+6i$

18. Multiplying the first equation by 5 and the second equation by -2 gives the equivalent system:

$$10x+15y=235$$
$$-10x+8y=18$$

Adding the equations gives 23y = 253, y = 11. Substituting y = 11 into either equation yields x = 7. Therefore the system has the unique solution (7,11)

19. Writing y = 13x + 7 means that y = f(x). Interchanging x and y results in the equation x = 13y + 7 where $y=f^{-1}(x)$. Solving for y we have 13y = x - 7 so that $f^{-1}(x)=\frac{1}{13}x-\frac{7}{13}$ and $f^{-1}(98)=\frac{98-7}{13}=7$

20. $f(g(x))=f(x^2+3x+1)=(x^2+3x+1)^2+5(x^2+3x+1)-8$
$=x^4+3x^3+x^2+3x^3+9x^2+3x+x^2+3x+1+5x^2+15x+5-8$
$=x^4+6x^3+16x^2+21x-2$

21. $\frac{8}{\sqrt[3]{2}}=\frac{2^3}{2^{1/3}}=2^{8/3}$ and $\log_2(2^{8/3})=\frac{8}{3}$ by the definition of $\log_2 x$ (In general, if the base b is a positive number other that 1 then $y=\log_b x$ if and only if $x=b^y$

22. $\log\left(\frac{x^3\sqrt{y}}{z^2}\right)=\log(x^3\sqrt{y})-\log(z^2)=\log(x^3)+\log(y^{1/2})-\log(z^2)$
$=3\log x+\frac{1}{2}\log y-2\log z$ using three properties of the logarithm.

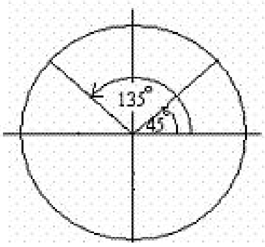
23. $y=\frac{2x+1}{3x-1}$, $y(3x-1)=2x+1$, $3xy-y=2x+1$, $(3y-2)x=y+1$,
$x=\frac{y+1}{3y-2}$.

24. $2(x+5)-3(3x-2)<5x+2$, $2x+10-9x+6<5x+2$,

$-7x + 16 < 5x + 2$, $-12x < -14$, $x > \frac{7}{6}$. When multiplying or dividing both sides of an inequality by a negative number, the direction of the inequality must be reversed.

25. Using the reference angle 45?,

$\sin(133?) \approx \sin(135?) = \sin(45?) = \frac{\sqrt{2}}{2} \approx 0.7$



26. Since AC is the hypotenuse of the triangle, the adjacent side AB = $\sqrt{10^2 - 6^2} = 8$. Therefore the cosine of $\angle BAC$ is 8/10 = 4/5 and its secant is 5/4.

27. The shaded region is the set of points that lie above the line y = -x + 1 and below the line y = x + 1. Therefore the set is $\{(x, y): y \geq -x + 1 \text{ and } y \leq x + 1\}$.

28. The amplitude of the function is $\frac{2-(-2)}{2} = 2$. Therefore a = 2. The period of the function is $\frac{2\pi}{3}$. Therefore b = 3.

29. Since θ is in the second quadrant, $\cos\theta < 0$ and $\cos\theta = -\sqrt{1-\sin^2\theta}$ $= -\sqrt{1-\frac{4}{9}} = -\frac{\sqrt{5}}{3}$. Therefore $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/3}{-\sqrt{5}/3} = -\frac{2\sqrt{5}}{5}$.

30. x = 3 is a vertical line, 3x+2y = 8 is a line with slope -3/2 and y = 0 is the x-axis. $x^2 - y^2 = 1$ is a hyperbola and $x^2 + y^2 = 1$ is a circle.

31. $3^{x+2} = 18$, $3^2 3^x = 18$, $3^x = 2$, $x = \log_3 2$

32. $x^5 + x^4 - 2x^3 = x^3(x+2)(x-1) < 0$ if x < -2 or 0<x<1.

x		-2		0		1	
$\mathbf{x^3(x+2)(x-1)}$	–	0	+	0	–	0	+